Limb Properties of Citrus as Criteria for Tree-Shaker Design

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Summary

THE vibrational characteristics of citrus limbs were measured at frequencies of 200 to 500 cpm to develop an analytical basis for designing inertia-type tree shakers for citrus. Limbs having diameters of 3.5 to 7.2 in. at the tree trunk were shaken.

A formula was developed for predicting the limb stroke of an inertia shaker if the crank throw, unbalanced mass, and boom mass of the inertia shaker and the apparent stiffness of the tree limb are known. Using a chart plotted for this formula, a shaker can be designed to shake a large range of limb sizes with approximately the same limb stroke.

Introduction

The development of inertia tree shakers for shaking citrus began in 1960. For most effective removal of citrus fruit, it is necessary to shake trees with a greater limb displacement and lower frequency (typically 300 cpm) than for shaker harvesting of other fruits, typically 1200 cpm (1)*. Formulas developed by Adrian and Fridley (2) for designing tree shakers to shake prune trees at relatively high frequencies were not valid for designing shakers to shake citrus at low frequencies. The measured limb displacement at the point of shaker attachment on a citrus tree was usually larger than the displacement calculated with Adrian and Fridley's formulas which were based on much higher shaking frequencies. The purpose of the study reported in this paper was to obtain information necessary for designing new inertia shakers that would effectively remove citrus by shaking.

Procedure

Six Valencia limbs, with diameters of 3.5 to 7.2 in. at the tree trunk, were shaken at two or three locations along the limb with a 4-in. shaker stroke and at one or two locations with a 6-in. shaker stroke. These locations were chosen so that there was one location on each limb which was shaken with both the 4 and 6-in. shaker stroke. The limbs were shaken with an inertia-type shaker (Fig. 1) which shook the limbs with an approximately sinusoidal shaking motion. An 8-channel oscillograph was used to simultaneously record the shaking force, acceleration, and displacement of each limb at the point of shaker attachment, and the displacement at three other locations along each limb. This data was recorded at shaking frequencies of 200 to 500 cpm.

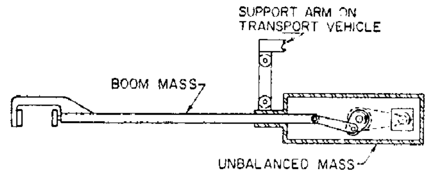

FIG. 1 Schematic diagram of inertial citrus shaker used in the study reported.

Instrumentation

The shaker boom was cut in two and a strain-gage force transducer inserted between the two halves of the boom. To calibrate the transducer, the acceleration and force signal were simultaneously recorded while shaking a 40-lb weight. The actual shaking force was calculated from the mass of the weight and its acceleration and used to calculate an appropriate calibration factor for the force transducer.

The net force applied to the limb was recorded by "electronic subtraction," *i.e.*, the force necessary to shake the limb clamp was electrically subtracted from the output of the transducer by using part of the acceleration signal from an accelerometer mounted on the clamp. This could be done since the force to shake the limb clamp (a pure mass) was directly proportional to the acceleration of the limb clamp.

A strain-gage accelerometer attached to the limb clamp was used to detect the motion of the limb at the point of shaker attachment. Piezoelectric accelerometers were attached to the limb at three other locations. Double-integrating vibration meters, with a frequency response down to 100 cpm, were used to integrate the accelerometer signals twice to obtain the displacement of each accelerometer. The displacement output signals from the double integrators were recorded with the 8-channel oscillograph.

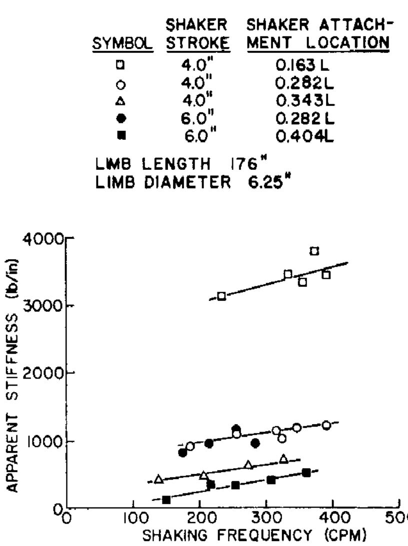

FIG. 2 Apparent stiffness of a citrus limb showing typical effect of shaking frequency.

Results

Apparent Stiffness

One method of describing the dynamic characteristics of a tree limb is to determine the ratio of the shaking force (F_L) to the displacement (S_L) at the point of shaker attachment. This ratio is termed the apparent stiffness (K). The apparent stiffness has been found to be particularly significant for describing the dynamic characteristics of citrus limbs because they are more closely approximated by a spring than by a mass.

Apparent stiffness of a typical limb (limb B, the second largest limb tested) is plotted as a function of shaking frequency (Fig. 2). The apparent stiffness of each limb was the same for both the 4-in. and 6-in. shaker stroke when the same point of attachment was used. This is to be expected since apparent stiffness is supposed to be a function of the structure shaken and should not depend on the characteristics of the shaker being used to shake it. The apparent stiffness of all but the largest limb increased slightly with increasing frequency. The apparent stiffness of the largest limb was constant with changing frequency. A larger percentage increase in the apparent stiffness with increasing frequency was noted for small limbs than for the large

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* Numbers in parentheses refer to the appended references.

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limbs. The further the point of shaker attachment from the tree trunk, the greater the per cent increase in apparent stiffness as the shaking frequency was increased. A small change in the apparent stiffness with changing frequency indicates the limb might be approximated by a spring. This will be discussed in the next section.

Large limbs had a larger apparent stiffness than small limbs, and attaching the shaker closer to the trunk always increased the apparent stiffness (Fig. 3).

Shaking-Force Phase Angle

An important property of a tree limb is the phase angle of the shaking force relative to the displacement of the limb at the point of shaker attachment. The phase angle, together with the apparent stiffness, completely describes the dynamic properties of a limb as the shaker "sees" it. The shaking force leads the displacement by a phase angle of 0 deg when vibrating a pure spring, and it leads by a phase angle of 90 deg when vibrating a dashpot (pure damping), and by 180 deg when vibrating a pure mass (3).

The phase angle (Θ) was calculated from the relative positions of the maximum oscillograph trace amplitudes for the shaking force and the limb displacement at the shaker attachment (Fig. 4).

The phase angle was always less than 90 deg (Fig. 5), indicating the limb characteristics are more like those of a spring than a mass in the frequency range tested, as previously noted in the discussion of apparent stiffness.

The phase angle tended to decrease slightly as the attachment point of the shaker was moved closer to the trunk. Limb size and shaker stroke had little effect on the phase angle.

Mode of Limb Vibration

Analysis of the displacement for the four accelerometer locations on each limb showed the general manner in which the limbs vibrated. The displacement at the accelerometer closest to the end of the limb lagged 100 to 200 deg behind the displacement of the accelerometer closest to the trunk. This indicated that the limbs were vibrating at approximately their second natural frequency. The end portion of a cantilever beam vibrating at its second natural frequency would be 180 deg out-of-phase with the remainder of the beam (4). However, on none of the six limbs were any nodes observed at any frequency. The displacement amplitude of the vibration measured by the accelerometer always increased towards the end of the limb. This indicates that citrus limbs are highly damped structures at the frequencies tested and that the vibration wave gradually increases in amplitude as it progresses to smaller and smaller wood and is finally dissipated by the leaves.

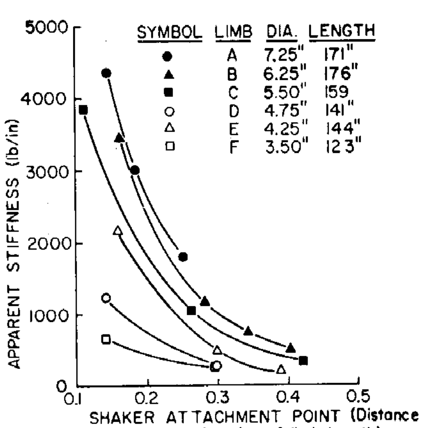

FIG. 3 Apparent stiffness of citrus limbs when shaken at 350 cpm.

Calculation of Limb Displacement from Measured Limb and Shaker Characteristics

The apparent stiffness (K) and phase angle (Θ) of the limb can be used to calculate the expected limb displacement amplitude (S_L) for an inertia-type tree shaker if the boom weight and unbalanced weight of the shaker are known, assuming the limb, shaker boom, and unbalanced weight all move with a sinusoidal motion.

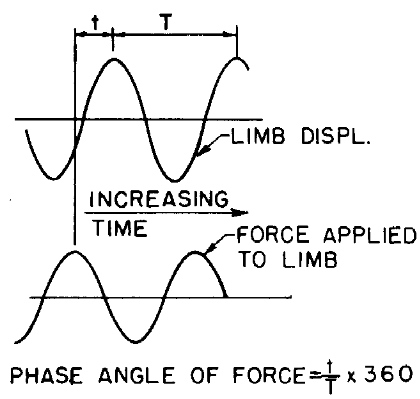

FIG. 4 Method of calculating shaking-force phase angle.

To derive an equation for limb displacement, vectors were used in a similar manner to the analysis of alternating-current circuits (5). The absolute value of a vector is the zero-to-peak amplitude of a quantity that is changing sinusoidally between plus and minus the peak amplitude. The phase angle indicates when the peak amplitude of the quantity occurs in relation to the maximum limb displacement at the point of shaker attachment.

The addition or subtraction of sinusoidal quantities of the same frequency (6) yields sinusoidal quantities of the same frequency. In the following derivation, sinusoidal quantities of the same units and frequency are added

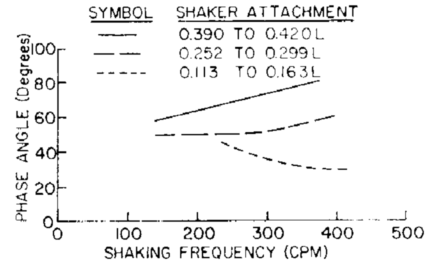

FIG. 5 Phase angle by which the shaking force led the limb displacement.

and subtracted vectorially to give the corresponding absolute value and phase position of the resultant vector. Each vector is actually a rotating vector and thus its position is a function of time, *i.e.*, equation [2] below written as a function of time would be:

$$\vec{F}_L = S_L K \cos(\omega t + \Theta) + j\, S_L K \sin(\omega t + \Theta).$$

However, the term ωt does not affect the addition or subtraction of two vectors and the solution is not dependent on the value of ωt. To simplify the equations, ωt is set equal to zero as is the convention in alternating current analysis (6).

Vectorially adding the force to shake the boom and the force to shake the limb will give the force exerted on the boom by the unbalanced weight. Expressed in equation form:

$$\vec{F}_u = \vec{F}_L \mathbin{+\!\!\rightarrow} \vec{F}_b \quad \text{. . . . . . . [1]}$$

Writing the force to shake the limb as a function of the apparent stiffness of the limb and dividing it into its real and imaginary components, we obtain:

$$\vec{F}_L = S_L K \cos\Theta + j\, S_L K \sin\Theta \quad \text{. . . . . [2]}$$

The displacement of the shaker boom is equal to the displacement of the limb. From basic vibration theory, the acceleration of the boom is its displacement times the frequency (ω) squared with the acceleration lagging the displacement by 180 deg indicated by a minus sign. Consequently, the force to shake the shaker boom may be written as:

$$\vec{F}_b = -M_b \vec{S}_L \omega^2 \quad \text{. . . . . [3]}$$

Substituting equations [2] and [3] into equation [1], the force exerted by the unbalanced weight is:

$$\vec{F}_u = S_L K \cos\Theta - M_b S_L \omega^2 + j\, S_L K \sin\Theta \quad \text{. . . . . [4]}$$

The force exerted by the unbalanced weight may also be determined from the mass of the weight, the displacement amplitude of the weight, and the shaking frequency from the equation:

$$\vec{F}_u = M_u \vec{S}_u \omega^2 \quad \text{. . . . . [5]}$$

This equation is obtained in the same

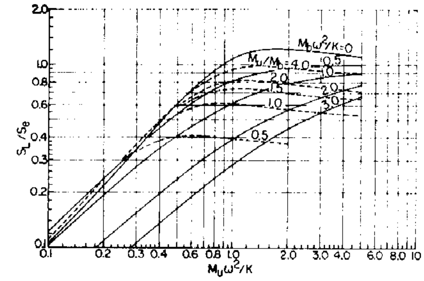

FIG. 6 Effect of limb and shaker characteristics on limb displacement.

manner as equation [3]. The difference in sign is because equation [5] is the force exerted *by* the unbalanced weight while equation [3] is the net force exerted *on* the boom.

Solving for S_u:

$$\vec{S}_u = \frac{\vec{F}_u}{M_u\omega^2} \quad [5a]$$

Substituting equation [4] into equation [5a], equation [6] for the displacement of the unbalanced weight is obtained.

$$\vec{S}_u = \frac{S_L K \cos\Theta - M_b S_L\omega^2}{M_u\omega^2} + \frac{j S_L K \sin\Theta}{M_u\omega^2} \quad [6]$$

The displacement of the boom plus the displacement of the unbalanced weight with respect to the boom must equal the displacement amplitude of the unbalanced weight. Expressed in equation form:

$$\vec{S}_b + \rightarrow \vec{S}_e = \vec{S}_u \quad [7]$$

Substituting $\vec{S}_b = \vec{S}_L$ and rearranging:

$$\vec{S}_e = \vec{S}_u \rightarrow \vec{S}_L \quad [7a]$$

Substituting equation [6] for $\vec{S}_u$ in equation [7a] and simplifying:

$$\vec{S}_e = \frac{S_L}{M_u\omega^2}(K\cos\Theta - M_b\omega^2 - M_u\omega^2 + j K \sin\Theta) \quad [8]$$

The absolute value or peak amplitude of S_e is:

$$S_e = \frac{S_L}{M_u\omega^2}\sqrt{(K\cos\Theta - M_b\omega^2 - M_u\omega^2)^2 + (K\sin\Theta)^2} \quad [9]$$

Solving for S_L,

$$S_L = \frac{S_e M_u\omega^2}{\sqrt{(K\cos\Theta - M_b\omega^2 - M_u\omega^2)^2 + (K\sin\Theta)^2}} \quad [9a]$$

A dimensionless form of equation [9a] is plotted in Fig. 6 for a phase angle of 55 deg. Using Fig. 6, the effect of a change in the shaker or limb properties can be readily determined. Fig. 7 gives a correction factor for phase angles other than 55 deg.

The correction factor is plotted from the equation:

$$\text{Correction factor} = \frac{\sqrt{[\cos 55° - (M_b + M_u)\frac{\omega^2}{K}]^2 + (\sin 55°)^2}}{\sqrt{[\cos\Theta - (M_b + M_u)\frac{\omega^2}{K}]^2 + (\sin\Theta)^2}}$$

This solution assumes that the shaker is powered with a motor large enough to drive it at the frequency it was designed for. A general formula for the power to drive a shaker cannot be given, since each shaker design would be different. However, the horsepower (P) dissipated in the limb is a function of the limb properties, limb displacement, and frequency with which the limb is being shaken and is given by the formula:

$$P_{avg} = 7.92 \times 10^{-6} S^2_L K f \sin\Theta \quad [10]$$

Use of Limb Properties and Fig. 7 For Shaker Design

Any shaker with a constant unbalanced weight and constant boom weight will operate along one of the family of lines in Fig. 6 labeled M_u/M_b. If the frequency of operation and apparent stiffness of a limb are known, in addition to the boom weight and unbalanced weight of the shaker, the S_L/S_e ratio can be determined. The limb displacement can be determined from the S_L/S_e ratio and the stroke of the shaker.

To illustrate the use of Figs. 6 and 7, the operation of two different shakers will be compared. An experimental shaker which effectively removed citrus fruit had the following properties:

Approximate frequency of operation	350 cpm
Weight of shaker boom	170 lb
Unbalanced shaker weight	230 lb
Throw of crankshaft	3 in.

Converting the above to standard units;

$$\omega = \frac{2\pi f}{60} = 36.6 \text{ rad/sec}$$

$$M_b = 0.440 \frac{\text{lb sec}^2}{\text{in.}}$$

$$M_u = 0.595 \frac{\text{lb sec}^2}{\text{in.}}$$

$$S_e = 3 \text{ in.}$$

The displacement ratio (S_L/S_e) will be found for three sizes of limbs. The ratio M_u/M_b is 1.35; therefore, this shaker will operate along a line slightly below the dashed line labeled 1.50 in Fig. 6. The apparent stiffness (K) can be approximated from Fig. 3 for a shaking frequency of 350 cpm. For a 6¼-in. limb shaken at 0.27L point of attachment, K equals 1200 lb/in.

The dimensionless ratio $M_u\omega^2/K$ is calculated to be 0.665 and from Fig. 6; S_L/S_e is determined to be 0.66. The values of S_L/S_e were determined in a similar manner for K = 800 lb/in. and 400 lb/in. and are shown in Table 1. The peak-to-peak displacement of the limb is determined by multiplying the ratio S_L/S_e by the stroke of the shaker ($2 S_e$).

The phase angle in these examples was assumed to be 55 deg. If the phase angle was some value other than 55 deg, the values of S_L/S_e from Fig. 6 would be multiplied by a correction factor from Fig. 7. For phase angles between 50 and 60 deg – the usual range for citrus limbs – the uncorrected values of S_L/S_e from Fig. 6 are correct within ± 10 per cent.

The limb displacement was calculated for an experimental shaker that did not effectively remove citrus fruit and was particularly ineffective when used to shake large limbs. The characteristics of this shaker were as follows:

Approximate frequency of operation	350 cpm
Weight of shaker boom	128 lb
Unbalanced shaker weight	92 lb
Throw of crankshaft	3 in.

The limb stroke produced by this shaker was greatly reduced when the apparent stiffness of the limb was greater than 800 lb per in (Table 1), which explains the ineffectiveness of this shaker when shaking large limbs. The crankshaft throw of this shaker could be increased until it effectively shook the large limbs but then it would shake the small limbs too vigorously.

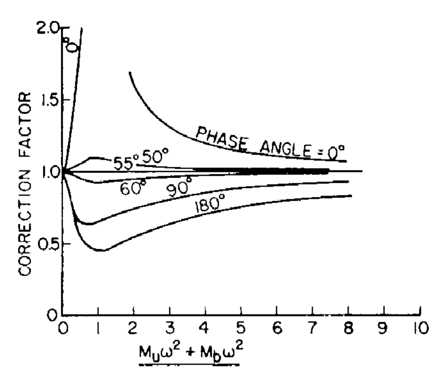

FIG. 7 Correction factor for limbs with phase angles (Θ) other than 55 deg.

verall Shaker Operation

Any conventional inertia shaker with constant boom weight and unbal-nced weight will operate along a con-tant M_u/M_b ratio line (Fig. 6). The exact point on the constant M_u/M_b ratio line is determined by the appar-ent stiffness (K) of the limb being shaken and the frequency at which it is being shaken. Locating the region of the M_u/M_b curve along which a proposed shaker design will operate, for the limb sizes to be shaken, helps to know how the design might best be improved.

If a shaker operates along the por-tion of the M_u/M_b curve which has a steep slope, it will shake small limbs violently while not effectively shaking the large limbs. In this case, there would be a lot of potential gain by in-creasing the unbalanced weight. Gen-erally, the greater the unbalanced weight, the larger the limb displace-ment. However, increasing the unbal-anced weight beyond the steep portion of the $M_b\omega^2/K$ curve increases the limb displacement only slightly.

A light boom weight is desirable, but a compromise must be made since the boom weight cannot be made zero. Fig. 6 is helpful in obtaining a reason-able compromise. If the M_u/M_b ratio of a proposed design is 4.0 or more, very little can be gained by further re-ducing the boom weight. If the ratio is less than 1.0 or 0.5, the shaker ef-fectiveness would be greatly increased if the boom weight could be decreased.

TABLE 1. LIMB DISPLACEMENT OF TWO EXPERIMENTAL INERTIA SHAKERS CALCU-LATED FOR THREE DIFFERENT LIMB SIZES USING THE SHAKER CHARACTERISTICS AND FIG. 7

	K	S_L/S_e	Peak-to-peak limb displ
	An effective inertia shaker		
Large limb	1200	0.66	3.96
Medium limb	800	0.72	4.32
Small limb	400	0.70	4.20
	An ineffective inertia shaker		
Large limb	1200	0.31	1.81
Medium limb	800	0.44	2.64
Small limb	400	0.51	3.06

List of Symbols

A vector quantity is indicated by placing an arrow, "$\rightarrow$," over the symbol.

A plain symbol such as F_b, indicates the zero-to-peak magnitude of a quan-tity.

Symbol	Description
F_b,	force to shake the boom of a shaker without any limb attached, lb
F_L,	force applied to a limb by shaker boom, lb
F_u,	force exerted on the shaker boom by the unbalanced weight of an inertia shaker, lb
f,	shaking frequency, cpm
j,	a vector operator which rotates a vector 90 deg counterclockwise when applied as a multiplying fac-tor
K,	apparent stiffness of a limb ex-plained in text, lb per in.
L,	length of limb from trunk to poi[nt] where limb diameter decreased to ½ in.
M_b,	mass of the shaker boom, $\frac{(\text{lb-sec}^2)}{\text{in.}}$
M_u,	unbalanced mass of shaker, $\frac{(\text{lb-sec}^2)}{\text{in.}}$
S_e	displacement of the unbalanced mass relative to the limb (or shaker boom) displacement. On a crankshaft-type shaker, S_e is the crank throw, in.
S_L,	displacement amplitude of the limb. This is also used as the dis-placement amplitude of the shaker boom since it is rigidly attached to the limb, in.
S_u,	displacement amplitude of the unbalanced weight, in.
t,	time, sec.
Θ,	phase angle of F_L with respect t[o] S_L
ω,	shaking frequency, rad per sec

References

1 Lenker, D. H. and Hedden, S. L. Optim[um] shaking action for citrus fruit harvesting. AS[AE] Paper No. 66-132, ASAE, St. Joseph, Mi[ch.] 49085, 1966.

2 Adrian, P. A. and Fridley, R. B. Dynam[ics] and design criteria of inertia-type tree shak[ers.] Transactions of the ASAE 8:(1)12-14, 1965.

3 Jacobsen, L. S. and Ayre, Robert S. En[gi]neering Vibrations, chapter 5. McGraw-Hill B[ook] Co., Inc., 1958.

4 *Ibid.*, Chapter 10.

5 Fundamentals of AC and AC Circuit A[nal]ysis. Philco Techrep Division, Philco Corp., [Phil]adelphia, Pa., 1960.

6 Robertson, B. L. and Black, L. J. Ele[ctric] Circuits and Machines, 2nd edition, chapte[r] D. Van Nostrand Co., Inc., 1957.