FRACTAL ANALYSIS OF YIELD MAPS

R. Ehsani, D. Karimi, K. H. Lee

ABSTRACT. *Agricultural crop yield data usually are highly nonlinear and complex. Basic mathematical and statistical techniques are sometimes insufficient to describe the nature, trend, or cause of variations in yield. This article investigates fractal analysis of agricultural yield maps and describes a method of applying fractal analysis to multiple years of yield data. It also shows how patterns of yield variation can be described by fractal geometry. Crop yield was measured and mapped for an agricultural field for five consecutive years. In order to obtain a sufficiently dense set of points necessary for valid fractal analysis, a method was proposed to transfer the data points from* $\Re^3$ *to* $\Re^2$*. Analysis of the resulting data set revealed multifractality of the yield variations. It was shown that multifractal measures such as the Rényi spectrum can be used to quantify and compare global and local yield variations.*

Keywords. *Fractal analysis, Multifractals, Precision agriculture, Spatial variability, Yield map.*

Analyzing and interpreting yield data is one of the main steps in successful application of precision agriculture. Assessment and interpretation of yield data can be very difficult because sometimes there are not enough data on the soil or plant factors that cause yield variability. Most of the common attempts to characterize in-field spatial yield variability have used basic statistical measures or geostatistical methods (Eghball et al., 1999; Blackmore et al., 2003). Because in-field variations are highly non-linear and complex, these methods cannot fully describe the variation patterns.

Maps of crop yield variability have been studied more than any other type of variability map because crop yield maps show the final outcome of management decisions and indicate overall field variability better than other variability maps. Spatial variability in crop yield is the result of interactions among many factors, including topography of the terrain, soil physical and chemical properties, and soil water content. These factors generate variations at nested scales that may result in self-similar patterns of variation (Green and Erskine, 2004). If so, these variation patterns can be described by fractal geometry (Peitgen et al., 2004) and quantified by fractal dimensions or fractal dimension spectra. Moreover, using methods such as joint multifractal analysis, the variability of crop yield can be related to other spatial variables of interest (Meneveau et al., 1990).Knowledge of scale-dependent variation can also be useful when generalizing analysis results or management decisions from a smaller to a larger scale or vice versa (Bekele et al., 2005).

A few studies have used fractal analysis to characterize temporal and spatial variability in an agricultural field. Perfect and Blevins (1997) used fractal analysis of soil structure to compare different tillage practices. Evaluation of the effect of variable-rate fertilizer application strategies has also been completed using fractal analysis of soil nitrate (Eghball et al., 1999). Fractal analysis of temporal and spatial variability in yield data has shown that temporal (year-to-year) variability in crop yield might be more significant than any spatial variability (Eghball and Varvel, 1997). Multifractal and joint multifractal analysis methods have been used to investigate the variability of crop yield with terrain slope (Kravchenko et al., 2000). Furthermore, fractal dimensions have been used to quantify the variability in crop yield and near-surface soil water (Green and Erskine, 2004) and to identify the scales of variation of soil electrical conductivity (Bekele et al., 2005). Fekete (2001) computed fractal dimension of crop yield by a time series analysis of yield monitor output and the threshing cylinder torque.

The objectives of this study were to introduce an appropriate methodology for fractal analysis of agricultural yield maps and to demonstrate how the developed method can be used to quantify and compare global and local variations in crop yield.

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FRACTAL GEOMETRY

SELF-SIMILARITY AND FRACTALS

The word "fractal" was coined by Benoit B. Mandelbrot in 1975 (Mandelbrot, 1977) and refers to a rough or fragmented geometric shape or a quantity that displays self-similarity on several scales. In other words, a fractal is a geometric shape or a pattern of variation that can be divided into parts, each of which is, at least approximately or statistically, a scaled-down copy of the whole. Mandelbrot (1983) formally defines a fractal as "a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological di-

