Moisture Movement in a Porous, Hygroscopic Solid

J. D. Whitney and J. G. Porterfield
Assoc. Member ASAE   Member ASAE

THE methods of water removal from agricultural crops have been the subject of many studies since the advent of environmental control during drying. Most of this research has been devoted to bulk or batch drying. Little emphasis has been placed on how moisture moves within individual units or kernels. Such research is needed to understand better the moisture movement which occurs during the drying and curing of crops.

Over much of the moisture content range encountered in curing and drying, a kernel will shrink as water is removed. In this process, a moisture gradient is established with the outermost layers of the kernel becoming the driest. The outer layers thus tend to shrink more causing tensile stresses parallel to the surface. These stresses in the outer layers increase as the moisture gradient increases. Kernel breakage results if the magnitude of the moisture gradient is such that the outer layers are stressed beyond their ultimate strength. Large moisture removal rates produced by present curing methods result in increased moisture gradients and kernel breakage.

Dielectric or internal heating has been proposed as a means by which drying and curing might be accomplished. This technique might provide a means of controlling the moisture gradient for given moisture-removal rates during the curing process. The temperature at the center of the kernel would be higher than at the surface. This is in contrast to conventional means of applying heat, as in controlled environment bulk curing, in which case the temperature at the center is lowest. The temperature gradient in a dielectrically heated material favors a moisture-driving potential towards the surface since water-vapor pressure increases with temperature. The moisture removal rate can be increased with the use of this driving potential. However, the benefits of the potential cannot be fully employed unless simultaneous use of the potential can be made to decrease the moisture gradient. Therefore, the influence of a temperature gradient as a moisture-driving potential was investigated as a means of controlling the moisture gradient during the drying process.

Paper No. 67-342 presented at the Annual Meeting of the American Society of Agricultural Engineers meeting jointly with the Canadian Society of Agricultural Engineering at Saskatoon, Sask., Canada, June 1967, on a program arranged by the Electric Power and Processing Division. Approved as Journal Manuscript No. 1577 of the Oklahoma Agricultural Experiment Station.

The authors—J. D. WHITNEY and J. G. PORTERFIELD—are assistant agricultural engineer, Citrus Experiment Station, Lake Alfred, Fla., and professor of agricultural engineering, Oklahoma State University.

* Numbers in parentheses refer to the appended references.

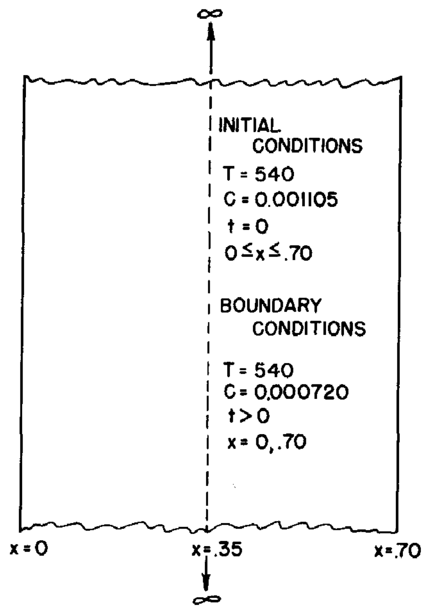

FIG. 1 Initial and boundary conditions of one-dimensional solid.

The senior author has presented a literature review on some investigations dealing with effects of temperature gradients on moisture transfer in porous solids (5)*. In general, the investigations revealed a net moisture transfer in the direction of decreasing temperature.

Theory

When the moisture of a hygroscopic solid changes, water must move from all parts of the solid to or from the surface. The mode by which water is transferred is not completely known. The transfer mode assumed in the following discussion is that of water-vapor diffusion through the pore spaces of the solid.

Consider a hygroscopic solid in equilibrium with its surrounding pore spaces. An increase or a decrease in the water-vapor concentration in the pore spaces causes the solid to absorb or evolve water, respectively. Absorption or evolution of water by the solid results in evolution or absorption of heat, respectively. Finally, this heat diffuses through the solid, causing changes in temperature, which affects the ability of the solid to absorb or evolve water. Thus, the transfer of moisture and the transfer of heat are coupled together and, in general, should be considered simultaneously.

To describe the effect of simultaneous diffusion of heat and water vapor in a hygroscopic solid, a mathematical treatment similar to that first used by Henry (3) is presented. The amount of water absorbed by the solid is assumed to have a linear dependence on the temperature of the solid and the water-vapor concentration in the pore spaces such that

$$M = c + aC - bT \quad \text{. . . . [1]}$$

where

M = quantity of moisture absorbed by the solid, lb H_2O per lb dry solid
C = concentration of water vapor in the pore spaces, lb H_2O per cu ft
T = absolute temperature of the solid, deg R
a, b, c = constants.

Equation [1] also describes the ability of the solid to evolve water.

The diffusion process of water vapor in the spaces and the solid is assumed to proceed according to Fick's First Law. The amount of water absorbed by an element of volume is equivalent to the increase of vapor concentration in the space plus the increase of water in the solid. Expressed mathematically, the movement of vapor is given by

$$D_s \frac{\partial^2 C}{\partial x^2} = f \frac{\partial C}{\partial t} + (1 - f)\, d_s \frac{\partial M}{\partial t} \quad \text{. . . . . [2]}$$

where

f = fraction of total volume occupied by the mixture of water vapor and air
$1 - f$ = fraction of total volume occupied by the solid
d_s = density of dry solid, lb per cu ft of solid
t = time, hr
D_s = coefficient of diffusion of water vapor in the solid, sq ft per hr
x = coordinate of unidirectional vapor movement, ft
∂ = partial derivative.

Consider the one-dimensional flow of heat within the solid in the x direction. The rate of temperature change in a volume element is controlled by

This article is reprinted from the TRANSACTIONS of the ASAE (vol. 11, no. 5, pp. 716, 717, 718, 719 and 723, 1968), the Transactions of the American Society of Agricultural Engineers, Saint Joseph, Michigan

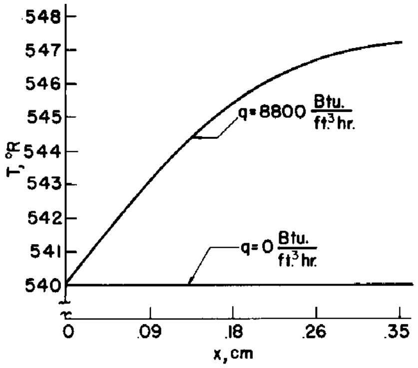

FIG. 2 Temperature distributions in one-dimensional solid.

the (**a**) heat conduction through the spaces and solid, (**b**) heat exchange as a result of water evaporation or absorption by the solid, and (**c**) heat generation applied by an external source. The mathematical expression describing the heat conduction is

$$c_s d \frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial x^2} + hd\frac{\partial M}{\partial t} + q \quad \ldots\ldots [3]$$

where

- c_s = specific heat of solid, Btu per lb deg F
- K = overall thermal conductivity of the solid and spaces, Btu per hr ft deg F
- d = bulk density of spaces and solids, lb solid per cu ft of overall volume
- h = heat required to evaporate water into the spaces, Btu per lb H_2O
- q = uniform and constant heat-generation rate to simulate dielectric heating, Btu per hr cu ft.

Equations [1], [2], and [3] were based on the following assumptions:

1 The change in moisture content of the solid is linearly dependent on changes in temperature and vapor concentration (Equation [1]).

2 The solid undergoes small changes in moisture content and temperature (Equation [1]).

3 The solid and adjacent spaces attain instantaneous equilibrium (Equation [1]).

4 The volumes occupied by the spaces and solids remain constant (Equation [2]).

5 Capillarity does not influence the movement of moisture within the spaces (Equation [2]).

6 D_s, K, c_s and d are constants (Equations [2] and [3]).

7 The heat associated with the loss or regain of moisture by the solid is the same (Equation [3]).

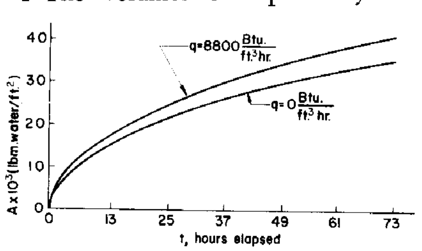

FIG. 3 Water loss from one surface of one-dimensional solid as a function of time.

The senior author has presented (5) the simultaneous solutions of equations [2] and [3] by eliminating M with equation [1]. Theoretical predictions were formulated from these solutions for moisture removal from an assumed, one-dimensional solid with a geometrical configuration of an infinite plate (Fig. 1). The finite dimension, 0.70 cm, corresponded to the x coordinate. Constants selected to characterize the solid in equations [1], [2], and [3] were:

f	=	0.01
d_s	=	71.0 lbm per cu ft
D_s	=	0.01 sq ft per hr
c_s	=	0.5 Btu per lbm deg F
d	=	70.0 lbm per cu ft
h	=	1000 Btu per lbm
k	=	0.08 Btu per hr ft deg F
a	=	153 cu ft per lbm
b	=	0.0019 deg R^{-1}
c	=	0.9959

With and without dielectric heating, the heat generation rate, q, was 8800 and 0 Btu per hr cu ft, respectively.

The initial conditions of C and T within the solid were 0.001105 (70 percent rh) and 540, respectively. After time zero, the boundary conditions of C and T at the surfaces were 0.000720 (46 percent rh) and 540, respectively. The initial and boundary conditions shown in Fig. 1 indicate that moisture removal from the solid was effected by only lowering the water vapor concentration at the surfaces.

Figs. 2, 3, 4, and 5 compare the results of the theoretical predictions with and without dielectric heating. Due to symmetry, only one-half of the solid was considered. Fig. 2 shows the temperature distributions after 1 hour of elapsed drying time. The respective distributions remained essentially unchanged up to 73 hours of elapsed drying time. Fig. 3 indicates that the water vapor loss, A, at the surface was always greater with heat generation. The greater water vapor loss with heat generation was the result of greater water vapor concentration gradient at any given elapsed drying time as illustrated in Fig. 4. The moisture distributions are shown in Fig. 5. With and without heat generation, the shape of the curves appeared to be the same. However, for given elapsed drying

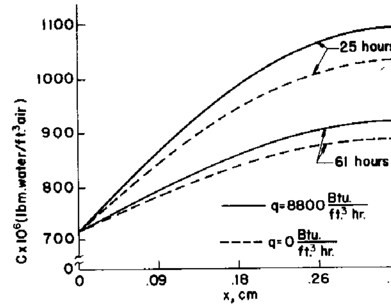

FIG. 4 Water-vapor concentration distributions in one-dimensional solid.

times, the steeper moisture gradients corresponded to the case with no heat generation. For given initial and boundary conditions, two effects were indicated. First, the amount of heat generation had negligible effects on moisture distribution for a given quantity of water loss from the surface. Second, heat generation decreased the time required to obtain a given moisture distribution and to remove a given quantity of water through the surface.

Methods, Materials, and Apparatus

One-dimensional drying experiments were set up to evaluate the validity of the theoretical predictions. A schematic of the system chosen for the drying experiments is shown in Fig. 6. Corn meal was selected as the test material because it was easy to handle and hygroscopic in nature. The glass tube, 11 mm ID and 8 in. long, maximized water-vapor flow along the length dimension of the sample. Rock wool and 2-in. thick styrofoam insulation insured approximate linear temperature gradients in the corn meal when T_0 and T_L were not equal.

Parameters believed to be important in the drying system are listed in Table 1. The parameters were formed into dimensionless groups which were treated as variables. According to the Buckingham Pi theorem, ten dimensionless groups should be formed when fourteen parameters are expressed in four dimensions. The dimensionless groups were identified by subscripts on P as follows:

$P_1 = M_x$

$P_2 = R$

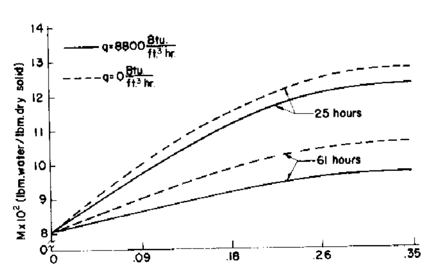

FIG. 5 Moisture distributions of one-dimensional solid.

TABLE 1. BASIC PARAMETERS IN THE DRYING SYSTEM

No.	Symbol	Parameter	Dimension*
1	L	Length of one-dimensional system, ft	L
2	T_o	Temperature at open end of system, deg R	Θ
3	T_L	Temperature at closed end of system, deg R	Θ
4	D	Diffusion coefficient for water vapor through the void spaces in the corn meal at 81 F, sq ft per hr	L^2T^{-1}
5	V_o	Vapor pressure of water vapor at open end of system, lbf per sq ft	FL^{-2}
6	M_i	Initial moisture content (dry basis) of corn meal (lbm per lbm) 100	O
7	x	Reference distance from open end of system, ft	L
8	M_x	Moisture content (dry basis) at x, (lbm per lbm) 100	O
9	t	Time elapsed, hr	T
10	k	First constant describing hygroscopic characteristics of kernel corn, 1 per deg R	Θ^{-1}
11	n	Second constant describing hygroscopic characteristics of kernel corn	O
12	V_s	Saturation vapor pressure at temperature at open end of system, lbf per sq ft	FL^{-2}
13	W	Molecular weight of water, lb per lb mole	O
14	R	Mass of water lost by system per dry unit mass of corn meal at time t, lbm per lbm.	O

* F, force; Θ, temperature; T, time, and L, length.

$$P_3 = n$$
$$P_4 = W$$
$$P_5 = T_o/T_L$$
$$P_6 = T_o k$$
$$P_7 = x/L$$
$$P_8 = M_i$$
$$P_9 = Dt/L^2$$
$$P_{10} = V_o/V_s$$

The thermal properties of corn meal was not included as parameters since unsteady-state conditions of temperature were almost nonexistent in the drying tests. T_o was held constant at 81 F. T_L ranged between 60 and 100 F. The diffusion coefficient, D, was measured as 0.31 sq ft per hr and was defined at 81 F because this represented the average temperature of the system for all drying tests. Parameters 10, 11, and 12 are those in Henderson's equilibrium equation (2) describing the hygroscopic characteristics of kernel corn. To restrict the scope of the study, P_3, P_4, and P_6 were held constant at 2.3, 18, and 0.002434, respectively. The apparatus and methods to control and/or to measure P_5, P_7, P_9, and P_{10} have been described by the senior author (5).

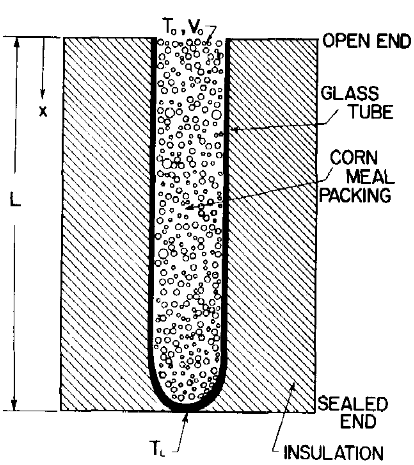

FIG. 6 Schematic of cross section of system used in the drying investigation.

In each drying test, a number of corn meal samples were prepared to obtain uniform moisture distributions at the desired initial moisture contents. Three of the samples were selected for determination of initial moisture distributions. Each sample was divided into eight equal parts. The moisture contents of each part were determined by the oven method. An average of all parts for the three samples yielded P_8. The remaining samples were weighed and placed in the drying apparatus. After the elapsed drying time specified by P_9, three samples were removed from the drying apparatus and weighed. The weight of water loss was the numerator of P_2. The denominator of P_2 was determined from the initial weights of the wet corn meal and P_8. Values of P_1 were determined by the oven method at 10 levels of P_7. These levels corresponded to the geometric center of finite lengths of the samples. They included four ½-in. sections adjacent to the open end and six 1-in. sections of the remaining length.

Table 2 shows an experimental schedule for the drying tests. Only one replication was made on each test. P_1 was observed as functions of P_5, P_7, P_8, P_9, and P_{10}. P_2 was observed as functions of P_5, P_8, P_9, and P_{10}. Note that data from test 1, $P_8 = 29.4$ were also used in test 3 and at intermediate levels of P_5 and P_{10} in tests 4 and 5, respectively.

Results

P_2 was found to increase with increasing P_8 and P_9 and decreasing P_5 and P_{10}. Generally P_1, for any given P_7, was found to decrease with increasing P_2.

Typical results of P_1 and P_2 values are the plotted points in Figs. 7 and 8, respectively. In each figure, the solid curves represents a prediction equation developed to describe the experimental results. The theoretical analysis suggested a series solution as a form of prediction equation for P_1. The final form of the equation selected to express P_1 was

$$\frac{P_1 - M_e}{P_8 - M_e} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin\,(2n-1)\,(P_7)\frac{\pi}{2}}{2n-1}\,[\exp\,(X)] \quad \ldots\ldots\ldots \quad [4]$$

where

$M_e = 3.6 + 16.3\, P_{10}$
$\exp\,(X) = e^X$
e = Naperian base of logarithms
$X = -[(2n-1)^2\,(C)\,(P_9)]\,[(1 - E\,(P_5 - 1)]\,[1 - F\,(P_{10} - 0.50)]$
C, E, F = constants

M_e represented the equilibrium moisture of corn meal at 541 deg R. For the range of P_{10} in the drying tests, 0.24 to 0.70, the equilibrium moisture content of shelled corn is approximately a linear function of P_{10} (1). The expression for X, except for the first

TABLE 2. EXPERIMENTAL SCHEDULE

Test No.	Observed	P_5	P_7		P_8	P_9	P_{10}
1	P_1, P_2	1.000	0.03125	0.4375	25.4	20.92	0.50
			0.09375	0.5625	29.8		
			0.15625	0.6875	35.0		
			0.21875	0.8125			
			0.3125	0.9375			
2	P_1, P_2	1.000	"		30.1	8.37	0.50
						16.74	
						25.11	
						33.48	
3	P_1	1.000	"		29.9	20.92	0.50*
4	P_1, P_2	0.966	"		29.9	20.92	0.50
		1.000*					
		1.040					
5	P_1, P_2	1.000	"		29.7	20.92	0.24
							0.50*
							0.70

* Used data from Test 1, $P_8 = 29.8$.

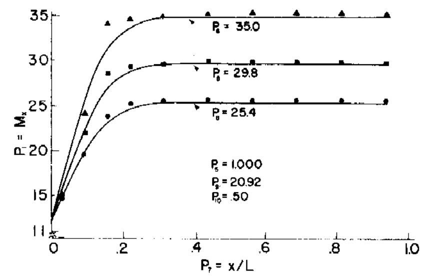

FIG. 7 P_1 versus P_7.

bracket, was suggested by the effect of P_5 and P_{10} on the drying rate.

Theoretically, if equation [4] yields the proper moisture distribution, P_1 vs. P_7, then the prediction equation for P_2 is

$$P_2 = \frac{1}{100}\int_0^1 P_8 \, d(P_7) - \frac{1}{100}\int_0^1 P_1 \, d(P_7)$$

or . . . . [5]

$$P_2 = G - G\frac{(8)}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}[\exp(X)]$$

where

$G = P_8 - 3.6 - 16.3 P_{10}$

Constants C, E, and F were evaluated by a computer. Their values were determined as 0.00073, 7.5, and 1.95, respectively.

Prediction equations [4] and [5] were tested for goodness of fit with experimental values with linear regression analysis. Predicted and experimental values were treated as dependent and independent variables, respectively. Tables 3 and 4 summarize the results. The values of r^2, coefficient of determination, were all 0.95 or greater. The "t" test (4) was used as the criterion to test the hypothesis that the value of b, slope of regression line, was one. The hypothesis was true for all values of b at the 0.01 level of significance.

The effect of a temperature gradient on the drying of cornmeal was evaluated using equations [4] and [5]. Moisture distributions, P_1 vs P_7, and water loss, P_2, were calculated at 12-hour intervals up to 72 hours of drying time. P_8 and P_{10} were 30.0 and 0.50, respectively. The two temperature gradients selected were $P_5 = 1.000$ and 0.966.

For any given elapsed drying time, P_2 was greater with $P_5 = 0.966$. After 48 hours of elapsed drying time with $P_5 = 0.966$, P_2 was 0.0230. P_2 was the same value after 60 hr of elapsed drying time with $P_5 = 1.000$. The moisture distributions, P_1 vs P_7, were identical for the two sets of conditions. Predictions presented in the theory section indicated similar results. That is, as the temperature of the solid decreases in the direction of moisture movement, the drying process is affected as follows: The drying rate is increased relative to the case of no temperature gradient, yet the moisture distributions are similar for given amounts of water removed from the solid. Therefore, for a given drying rate, a temperature gradient in the same direction of the moisture gradient favors a reduction in the moisture gradient, thus reducing the shrinkage stresses in the solid due to the moisture gradient.

Summary

High rates of water removal from agricultural crops usually result in steep moisture gradients and undesirable cracking in the individual units or kernels. New techniques of water removal are needed to increase drying rates without inducing intolerably high stresses which cause cracking.

Dielectric or internal heating was proposed as a means of inducing faster movement of water from porous, hygroscopic solids. A theory of moisture transfer in the vapor phase in a porous, hygroscopic solid was presented with and without internal heating. For given rates of water removal from the solid, the moisture gradient was reduced with internal heating.

Experimental results from the drying of one-dimensional samples of corn meal generally agreed with theoretical predictions. That is, relative to the case of no temperature gradient, the drying rate was greater with a temperature gradient in the same direction as the moisture gradient. For any given amount of water lost, the moisture gradients were comparable. This inferred that a temperature gradient could be used to increase the drying rate of a porous, hygroscopic solid without increasing shrinkage stresses associated with the moisture gradient.

Conclusions

The following conclusions, based on (a) the theory of moisture movement by the mechanism of vapor transfer and (b) the experimental results of the study, seem to be justified for the drying of porous hygroscopic solids:

1 After a given amount of water has been removed, the moisture gradients are similar whether or not a temperature gradient (resulting from internal heating) exists in the same direction as the moisture gradient.

2 Relative to the case of no temperature gradient, the drying rate is greater when a temperature gradient exists in the same direction as the moisture gradient. The greater drying rate is a result of a greater vapor concentration or vapor-pressure gradient.

3 A temperature gradient in the same direction as the moisture gradient can be used to decrease the total drying time, without increasing the shrinkage stresses in the solid due to the moisture gradient.

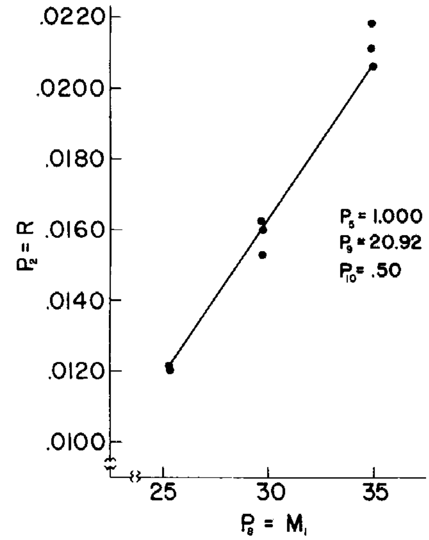

FIG. 8 P_2 versus P_8.

TABLE 4. SUMMARY OF b AND r^2 FOR COMPARISON OF PREDICTED AND EXPERIMENTAL VALUES OF P_2

Test No.	b	r^2
1	1.07	1.00
2	1.01	1.00
4	0.98	1.00
5	1.00	1.00

References

1 Hall, C. W., and Rodriguez-Arias, J. H. Equilibrium moisture content of shelled corn. **Agricultural Engineering** 33:(1) 29-32, January 1952.

2 Henderson, S. M. A basic concept of equilibrium moisture. **Agricultural Engineering** 33:(1) 29-32, January 1952.

3 Henry, P. S. H. Diffusion in absorbing media. Proceedings of Royal Society of London 171, Series A, 215-241, 1939.

4 Steel, R. F. D. and Torrie, J. H. "Principles and procedures of statistics." McGraw-Hill Book Co., Inc., New York, p. 171, 1960.

5 Whitney, J. D. Moisture transfer in a porous, hygroscopic solid. Unpublished Ph.D. thesis, Oklahoma State University, Stillwater, 1966.

TABLE 3. SUMMARY OF b AND r^2 FOR COMPARISON OF PREDICTED AND EXPERIMENTAL VALUES OF P_1

P_5	P_8	P_9	P_{10}	b	r^2
0.966	29.9	20.92	0.50	0.93	0.95
1.000	29.8	20.92	0.50	1.05	0.98
1.040	30.0	20.92	0.50	0.98	1.00
1.000	25.4	20.92	0.50	1.07	1.00
1.000	35.0	20.92	0.50	1.13	0.98
1.000	30.1	8.37	0.50	1.07	0.98
1.000	30.4	16.74	0.50	1.02	0.99
1.000	30.7	25.11	0.50	1.05	0.98
1.000	31.0	33.48	0.50	1.05	0.98
1.000	29.6	20.92	0.24	1.08	0.98
1.000	29.8	20.92	0.70	1.15	0.99