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DYNAMIC ANALYSIS OF A TRUNK SHAKER-POST SYSTEM

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ABSTRACT

Dynamic properties of a wooden post (viscous damping, mass, and stiffness) were measured and used along with corresponding characteristics of a shaker (unbalanced mass, total mass, eccentricity, and frequency of rotation) in the development of a mathematical model to predict applied shaker force and motion. Three lengths of a wooden post were cantilever supported in a concrete base structure and shaken with an inertia-type linear trunk shaker. Motion and force at the point of attachment were measured and analyzed. Predominate components of the system were generated force, total shaker mass, and post stiffness. Measured force and motion data were used to calculate an effective post stiffness. Force and displacement amplitudes and post stiffness predicted by the model agreed reasonably well with measured values.
KEYWORDS. Citrus, Modeling, Vibration, Shakers

INTRODUCTION

Tree shakers and their effects on trees have been researched for more than 25 years. The primary intent has been to establish criteria for the design of shakers which will remove fruit efficiently without damaging trees, although some researchers have investigated tree properties to design optimal dynamic input for mechanical harvesting. Some of the research has included model-based system analysis.

Fridley and Adrian (1960) modeled a tree limb as a cantilever and stated that the objective in removing fruit by shaking is to develop an inertial force on the fruit that exceeds the bonding force between it and the stem. They found that the force could be achieved best by shaking a tree limb at its natural frequency with a force applied to a point on the limb that is not a node (point of zero deflection). To establish criteria for design, Adrian and Fridley (1965) analyzed an inertia-type tree shaker attached to a limb as a single degree of freedom vibrating system. Using Newton's second law of motion, they derived a differential equation by summing forces on the shaker and limb and equating them to the inertial force of the system. They derived expressions for design force, torque, and power requirements of an inertia shaker and outlined a procedure for design.

Formulas developed by Adrian and Fridley (1965) for designing tree shakers to shake prune trees at relatively high frequencies (20 Hz) were not valid for designing shakers to shake citrus at lower frequencies (5 Hz). Thus, Lenker and Hedden (1968) measured and recorded the force and motion of a citrus tree limb and shaker (inertia-type) at the point of attachment and several other points along the limb. They determined the "apparent stiffness" (ratio of force to displacement) at several points along the limb. They found limb displacement to be a function of limb stiffness, shaker mass, and the magnitude and frequency of the applied force. Lenker and Hedden (1968) concluded that the reaction of a citrus tree limb to the shaking force was more nearly that of a spring than a mass and that viscous damping was present.

Hoag et al. (1971) investigated the internal and external damping of tree limbs. They determined internal or material damping with the logarithmic decrement method for specimens of almond wood mounted as cantilevers. To quantify internal damping, the log decrement method was used on the measured amplitudes of vibration after the specimens were excited primarily at their first natural frequency. External damping of tree limbs moving in air was determined by measuring the force of air friction on a limb attached to a moving truck.

Canavate et al. (1980) investigated the vibration characteristics of a tree shaker. They tested a shaker in the field on olive trees and also in the laboratory on a steel post embedded in concrete in the ground. They found the responses of trees of similar size and shape to vary widely, while the response of the post was consistent. They concluded that posts provide a convenient reproducible method of testing and characterizing a shaker.

Whitney et al. (1988) measured trunk shaker and citrus tree trunk motion simultaneously. They found amplitude of trunk motion to be correlated negatively with trunk circumference and positively with height above ground of the applied shaker force for four shaker modes.

Although much more research has been done on mechanical harvesting with trunk shakers, no literature has been found providing detailed dynamic analyses of a trunk shaker-tree system that relates physical properties of the tree and shaker to performance. As a first step in analyzing this complex system, we investigated the dynamics of a simpler system consisting of the trunk shaker and a wooden post mounted as a cantilever to simulate a tree trunk.

The main objective of this investigation was to determine the validity of a dynamic model of this trunk

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shaker-post system. The specific objectives of this research were to:

1. Design and construct a structure which would hold a wooden post as a cantilever for shaking;
2. Derive a mathematical model for predicting force and motion at the point of attachment of the shaker to the post;
3. Measure force and motion variables in the experimental shaker-post system;
4. Compare predicted and measured responses of the system.

METHODS AND EQUIPMENT

BASE STRUCTURE AND TEST POST

The base structure was constructed to hold a post as a vertical cantilever. It consisted of a steel tube 23 cm in OD with a 1 cm thick wall embedded in reinforced concrete (fig. 1) and 14 cm diameter steel collars at the top and bottom of the steel tube to secure an inserted post. One and one-half cubic meters of concrete were poured into the hole surrounding the steel tube.

SHAKER

A three-shaft, rotating mass, inertia-type linear trunk shaker was rigidly clamped to the wooden post (fig. 2). The mass of the shaker was 433 kg, excluding 65 kg of unbalanced masses. The unbalanced masses were rotated at a velocity ≤ 30 rad/s with 14 cm of eccentricity.

FORCE AND MOTION TRANSDUCER INSTRUMENTATION

A BLH C3P1* load cell mounted between the shaker and post clamp measured the shaking force applied to the post (fig. 2). An Action Pak 4251 amplifier increased the load cell voltage signal. A PCB 308B accelerometer accompanied by a PCB 480A battery power supply measured acceleration of the post near the point of the clamp attachment (fig. 2). Analog voltage signals of the accelerometer and load cell were simultaneously recorded on a TEAC R-61 cassette data recorder.

DATA PROCESSING SYSTEM

Recorded analog voltage signals from the force and motion transducers were sampled at 1 ms intervals and transferred to an Apple IIe microcomputer through a Cyborg Isaac 91A analog-to-digital converter.

A second-order bandpass digital filter in the computer software for data processing was used to reduce noise in the acceleration signal attributed to the shaker. Although not as noisy, the force signal was similarly filtered to retain the phase relationship between acceleration and force signals. Acceleration data were integrated numerically using the trapezoidal method (Stark, 1970) to obtain velocity and position over time.

Force, acceleration, velocity, and displacement data were stored on floppy disks for later use in analyses of the model and real, shaker-post systems. Mean values and standard deviations of the amplitudes of these variables were calculated with standard deviations indicating variability of n samples of the variables in a one second interval. Absolute error bounds of measured variables also were calculated by adding in each case the absolute error contributions of components in the relevant measurement system (Block, 1986).

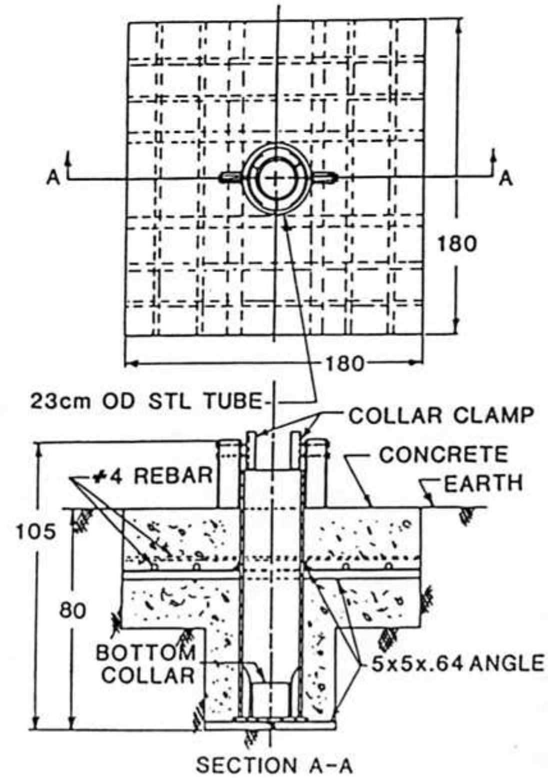

Figure 1–Base structure for holding wooden posts (dimension in cm).

PROPERTIES OF WOOD POST

The test post was a cypress log 14 cm in diameter (dimension D in fig. 2); its 210 cm length permitted the post to be mounted in the base structure and the shaker clamp to be attached at 3 heights, 84, 69, and 53 cm, above the upper collar of the base (shaker height L in fig. 2). The

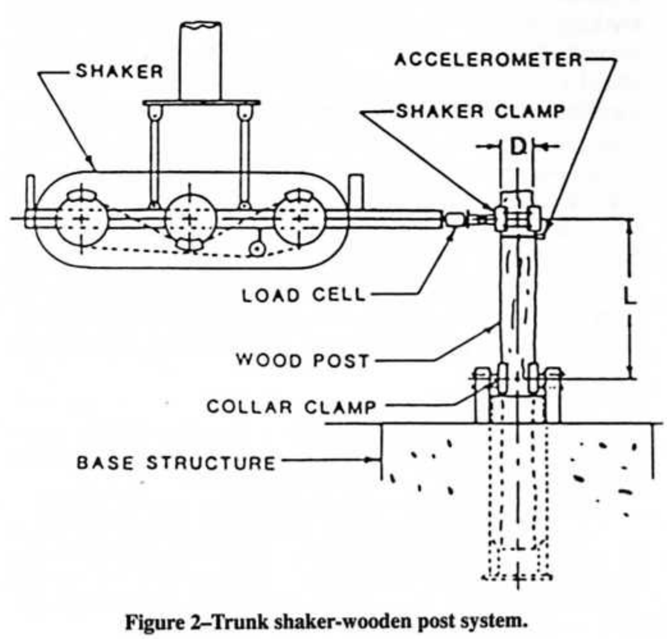

Figure 2–Trunk shaker-wooden post system.

*Mention of a tradename does not imply endorsement of the products by the University of Florida or Auburn University and is only provided as information for the reader.

mathematical model required mass, stiffness, and damping coefficient for each shaker height. The modulus of elasticity, an indication of material stiffness, was determined by measuring the deflection of a beam of uniform cross-section loaded as a cantilever. The mass of the post was determined from density (average of several cypress specimens) and measured volume above the base clamp. The damping coefficient and natural frequency of the test post were determined by fastening an accelerometer to the top of the post and striking it with a hammer (simulating an impulse) to cause it to vibrate. The accelerometer signal was recorded, integrated twice, and plotted versus time. The logarithmic decrement method (Morrill, 1957) was used to determine the damped natural frequency and damping coefficient.

Test Procedure

The post was secured in the base structure and its viscous damping coefficient was determined as described above. Then the shaker boom was clamped to the post at 84 or 69 or 53 cm above the base collar and operated at each of 3 constant velocities – 23, 24, and 27 rad/s – while acceleration and force data were recorded. The data were processed as indicated above, and force and displacement amplitudes were plotted vs. frequency.

Mathematical Model

Block (1986) derived a mathematical model for the shaker-post system diagrammed in figure 3 to express the motion of the point of shaker-post attachment in response to the applied force generated by the rotating, unbalanced shaker weights. The mathematical model for linear components is

$$M x''(t) + C x'(t) + K x(t) = M_u r \omega_2 \sin(\omega t) \qquad (1)$$

where

$x(t)$	=	displacement,
$x'(t)$	=	velocity, and
$x''(t)$	=	acceleration of the attachment point,

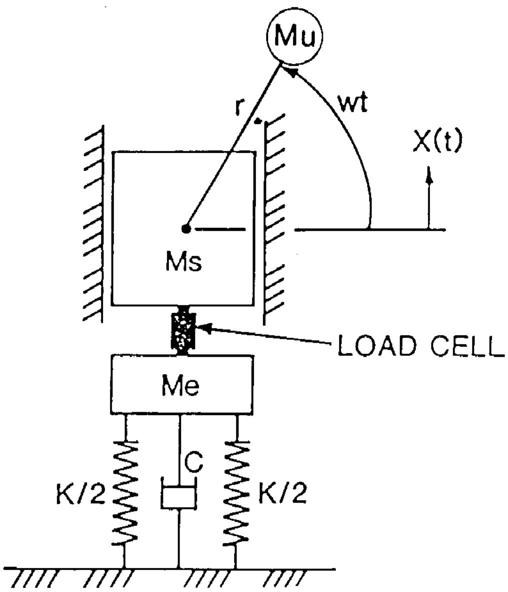

Figure 3–Model diagram depicting arrangement of load cell for measuring applied force to post.

t	=	time, and
the right hand member of eq. 1	=	the force generated by the shaker.

The amplitude of steady state displacement in response to this sinusoidal excitation is given by

$$X = \frac{M_u r \omega^2}{\left[\left(K - M\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}} \qquad (2)$$

where

X	=	steady state displacement amplitude, m,
M	=	$M_s + M_e$=total system mass, kg,
M_s	=	shaker mass, excluding rotating unbalanced mass M_u, kg,
M_e	=	equivalent post mass, kg,
M_u	=	rotating unbalanced mass, kg,
r	=	radius of eccentricity of M_u, m,
K	=	post stiffness, N/m,
C	=	viscous damping coefficient of the post, kg/s,
ω	=	angular velocity of rotating unbalanced masses, rad/s,
$\omega_n = (K/M)^{1/2}$	=	undamped natural frequency of shaker-post system, rad/s,
$\omega_{np} = (K/M_e)^{1/2}$	=	natural frequency of post alone, rad/s.

Of the total generated force (amplitude $F_g = M_u r\omega^2$), only the portion with steady state amplitude

$$F = \frac{\left[\left(K - M_e\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}}{\left[\left(K - M\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}} F_g \qquad (3)$$

$$= M_u r \omega^2 \frac{\left[\left(K - M_e\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}}{\left[\left(K - M\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}}$$

is applied to the mass, damping, and stiffness of the post and measured by the load cell; the remainder is applied to shaker mass M_s.

Transmissibility ratio T, defined by

$$T \equiv \frac{F}{F_g} = \frac{\left[\left(K - M_e\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}}{\left[\left(K - M\omega^2\right)^2 + \left(C\omega\right)^2\right]^{1/2}} \qquad (4)$$

measures the variation with frequency of applied force F in response to generated force F_g.

Results and Discussion

Properties of Wood Post

The post masses were determined to be 5.8, 7.3, and 8.6 kg for the 53, 69, and 84 cm shaker heights, respectively. However, due to arrangement of the post as a vertical cantilever, its effective, dynamic mass did not equal its static mass above the upper base clamp; it was the equivalent point mass at the free end of the cantilever. Effective masses of the post for the three shaker heights were calculated using Raleigh's method (Thomson, 1953), which accounts for dynamic displacement of a post varying from no amplitude at its base to maximum amplitude at the top. Effective masses of 1.36, 1.71, and 2.03 kg were calculated for the 3 heights (Table 1). These masses, in comparison to the 433 kg shaker mass, were considered to have negligible effect on force and motion of the shaker-post system. Post stiffnesses were calculated to be 3.67, 1.65, and 0.92×10^6 N/m, and damping coefficients were calculated to be 50.0, 22.1, and 18.3 kg/s for the three shaker heights. Calculated undamped natural frequencies of the shaker-post system ($(K/M)^{1/2}$) for the 53, 69, and 84 cm shaker heights were 92, 61, and 46 rad/s, respectively, while for the three post lengths alone ($(K/M_e)^{1/2}$) they were 1642, 982, and 673 rad/s, respectively. Note that all natural frequencies were at least 70% larger than the highest shake frequency. Viscous damping was found to have negligible effect on system dynamics. Amplitudes of damping forces were calculated to be less than 10 N, while amplitude of measured forces of the shaker was as high as 9.9 kN. In summary, the inertia and generated force of the shaker were counteracted almost entirely by stiffness of the post.

Displacement

Table 2 shows predicted and measured means, standard deviations, and absolute error bounds of displacement amplitude. Predicted and measured displacement amplitudes increased with frequency and shaker height. According to vibration theory, as shaking frequency approaches the natural frequency of a system, the amplitude of motion should increase sharply. This effect was most evident with the 84 cm shaker height, for which the sharpest increase in displacement amplitude was observed, a reflection of its natural frequency being nearer to the shaker frequency than those of the two shorter shaker heights. Measured amplitude increased more sharply with frequency than predicted values for the 69 and 84 cm shaker heights, indicating that natural frequencies probably were less than calculated values. This effect was not observed for the 53 cm shaker height, which had the highest natural frequency, far beyond the maximum shaking frequency.

Force

As shown in Table 3, predicted and measured force amplitude increased with frequency due to increased generated force, F_g, and increased transmissibility ratio, T. Table 4 lists model-predicted values of T. Since shake frequencies $\omega < \omega_n$ were employed, T would always be greater than one and would increase as ω approached ω_n.

TABLE 1. Wooden post characteristics used in mathematical model

Shaker height, L (c m)	Effective mass (kg)	Damping coefficient (kg/s)	Equivalent spring constant ($N/m \times 10^{-6}$)
53	1.36	50.0	3.67
69	1.71	22.1	1.65
84	2.03	18.3	0.92

TABLE 2. Steady state amplitudes of measured and predicted post displacement

Shaker height, L (c m)	Shaker frequency (rad/s)	Displacement aplitude* Predicted (m m)	Measured (m m)	Error bound (m m)	(n)	Std dev (m m)
53	23	1.4	1.3	1.2	7	0.36
53	24	1.6	2.2	1.3	8	0.75
53	27	2.0	2.2	1.3	9	0.55
69	23	3.4	3.0	1.3	7	1.15
69	24	3.9	3.7	1.3	8	1.84
69	27	5.0	5.7	1.4	9	1.30
84	23	7.3	6.4	1.5	7	3.09
84	24	8.4	8.0	1.5	8	4.40
84	27	11.5	18.6	1.9	9	7.09

* Measured values are means of n amplitudes measured over a one second time interval.

Theoretically, if the damping coefficient was zero, T would approach infinity as ω approached ω_n. As may have been expected from the low values of C, several posts (at least 84 cm long) were broken in preliminary trials when the shaker was operated at its maximum frequency of 40 rad/s.

As with displacement, measured forces in the 69 and 84 cm shaker height systems increased more sharply with frequency than predicted, indicating the natural frequencies of those systems may have been less than predicted by the model.

Stiffness

As mentioned earlier, inertia and viscous damping of each post at frequencies employed were negligible in comparison to its stiffness, and each post was expected to act essentially as a spring. To check this for each post, effective stiffness S = F/X was calculated and compared with corresponding ratios of experimental force and displacement amplitudes. From 2 and 3,

$$S = \left[\left(K - M_e\omega^2\right)^2 + \left(Cw\right)^2\right]^{1/2} \cong K \quad (5)$$

Measured and predicted values of effective stiffness are listed in Table 5. Predicted values were in every case higher than measured values and, in general, measured effective stiffness was found to decrease with increasing

TABLE 3. Amplitudes of measured and predicted steady state forces

Shaker height, L (c m)	Shaker frequency (rad/s)	Gen* force, F_g (N)	Applied force amplitude F† Predicted (N)	Measured (N)	Error bound (N)	(n)	Std dev of F (N)
53	23	4786	5158	4100	207	7	85
53	24	5325	5789	5000	216	8	242
53	27	6489	7192	5800	224	9	208
69	23	4786	5689	4500	211	7	397
69	24	5325	6467	4900	215	8	286
69	27	6489	8268	7000	236	9	126
84	23	4786	6700	5000	216	7	93
84	24	5325	7805	5600	222	8	141
84	27	6488	10588	9900	265	9	241

* Generated force, F_g, by unbalanced masses are predicted, not measured.

† Measured values are means of n amplitudes measured over a one second time interval.

frequency, while that predicted by 5 remained relatively constant (less than 0.1% change) over the range of frequencies employed. Measured effective stiffness decreased with increasing frequency for the two longer shaker heights, but decreased and then increased for the 53 cm height. As mounted in the base structure, the post did not act precisely as the simple cantilever assumed in the model, which could have caused stiffness to vary differently than predicted by 5. The post was clamped only at the bottom and top of the base structure (fig. 1), permitting it to flex between the clamps.

DAMPING

Table 1 shows values of the damping coefficient used in the mathematical model. With viscous damping coefficients determined experimentally to be 50, 22, and 18 kg/s for the three shaker heights and the velocity amplitudes predicted by the model to be 0.052, 0.132, and 0.306 m/s, damping force amplitudes of 2.6, 2.9, and 5.5 N were calculated for 53, 69, and 84 cm shaker heights, respectively, with the shaker operating at 27 rad/s. Thus, damping forces were very small compared to predicted and measured shake forces (Table 3), which ranged from 4.1 to 10.6 kN.

SUMMARY AND CONCLUSIONS

Force, displacement, and effective stiffness predicted by the model of a shaker-post system agreed reasonably well with measured values, but indicate a more complex model should be considered. Nevertheless, one is encouraged to model a tree and validate it with shake experiments on real trees. Useful experience might be gained first by modeling and shaking an artificial tree, consisting of trunk, branches, and simulated leaf mass, mounted as a compound cantilever in the base structure used in these experiments.

TABLE 4. Model predicted force transmissibility

Shaker height, L (cm)	Shaker frequency (rad/s)	System natural frequency (rad/s)	Frequency ratio	Transmissibility ratio
53	23	90	0.25	1.08
53	24	90	0.27	1.09
53	27	90	0.30	1.11
69	23	60	0.38	1.19
69	24	60	0.40	1.21
69	27	60	0.45	1.27
84	23	45	0.51	1.40
84	24	45	0.54	1.47
84	27	45	0.59	1.62

TABLE 5. Measured and predicted effective stiffnesses of test posts

Shaker height, L (cm)	Shaker frequency (rad/s)	Stiffness Predicted ($N/m \times 10^{-6}$)	Stiffness Measured ($N/m \times 10^{-6}$)	Error bound ($N/m \times 10^{-6}$)
53	23	3.67	3.13	3.05
53	24	3.67	2.29	1.52
53	27	3.67	2.63	1.65
69	23	1.65	1.50	0.72
69	24	1.65	1.33	0.52
69	27	1.65	1.23	0.34
84	23	0.92	0.79	0.22
84	24	0.92	0.70	0.16
84	27	0.92	0.53	0.07

The wooden post mounted as a cantilever in these experiments was a vibrating second order system, with mass, stiffness, and viscous damping, and provided first approximations to a tree. However, the post acted nearly as a pure spring at the shaker frequencies employed; its mass and damping force was negligible in comparison to stiffness. With this condition extant, the spring constant of a post can be estimated by the ratio of steady state force and displacement amplitudes measured at the point of shaker-post attachment. The natural frequency of each shaker-post system was approximately equal to the square root of post stiffness divided by shaker mass because shaker mass was much larger than post mass. With a real tree, top masses of limbs and leaves would be substantial and relative importance of parameters and the system dynamics would change significantly.

In these experiments, natural frequencies of shaker-post systems tested were 80 to 95% less than natural frequencies of the posts alone due to the dominant shaker mass, and shake frequency was well below all natural frequencies. Steady state displacement and force amplitudes increased with shaker height because natural frequencies of the shaker-post systems decreased with increased effective length of the constant radius posts. If a shaker were capable of sufficiently high frequency, the natural frequency of a shaker-post system could be estimated by observing the frequency at which force and displacement maximize.

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